1) Let A, B, C and D be four distinct points such that $\overrightarrow{OA} = 3\underline{i} + 2\underline{j} + 3\underline{k}$, $\overrightarrow{OB} = 2\underline{i} + \underline{j}$, $\overrightarrow{OC} = 4\underline{j} + 4\underline{k}$ and $\overrightarrow{OD} = 7\underline{i} + 4\underline{k}$, where O is the origin.

a) Let α be the plane passing through the points A, B and C. Find the equation of α in scalar form.

b) Let l be the straight line passing through A and the midpoint of the line segment BC. Find the equation of l in scalar form.

c) Let β be the plane passing through the point D and is perpendicular to the line l. Find the equation of β in scalar form.

d) Calculate the angle between α and β.

1)

Let $\underline{a} = \underline{i} - \underline{j}$, $\underline{b} = \underline{i} + \underline{j} + \underline{k}$ and $\underline{c} = 4\underline{i} + 2\underline{j} + 3\underline{k}$ be vectors; O is the origin and suppose $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$, and $\overrightarrow{OC} = \underline{c}$.

a) Find the area of the triangle ABC.

b) Show that the points $O, A, B, and\ C$ are on a plane.

c) Find equation of the plane go through the points $O, A, B, and\ C$.

1) Let A, B, C and D be four distinct points such that $\overrightarrow{OA} = 3\underline{i} + 2\underline{j} + 3\underline{k}$, $\overrightarrow{OB} = 2\underline{i} + \underline{j}$, $\overrightarrow{OC} = 4\underline{j} + 4\underline{k}$ and $\overrightarrow{OD} = 7\underline{i} + 4\underline{k}$, where O is the origin.

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d) Calculate the angle between α and β.

1. (a) Let A, B, C and D be points with coordinates $(4, 5, 1), (0, -1, -1), (3, 9, 4)$ and $(-4, 4, 4)$. Use **vector methods** for the following problems.

   i. Find the vector equation and cartesian equation of the straight line which passes through A and parallel to $\overrightarrow{BC}$.

   ii. Find the area of the triangle BCD.

   iii. Show that A, B, C and D are coplanar.

   iv. Find the vector equation and cartesian equation of the plane $ABCD$.

   (b) Let $\underline{a}, \underline{b}$ and $\underline{c}$ be non zero vectors. Show that,

   i. $[\,\underline{a}+\underline{b}\,,\underline{b}+\underline{c}\,,\underline{c}+\underline{a}\,] = 2[\,\underline{a}\,,\underline{b}\,,\underline{c}\,]$

   ii. $\underline{i} \times (\underline{a} \times \underline{i}) + \underline{j} \times (\underline{a} \times \underline{j}) + \underline{k} \times (\underline{a} \times \underline{k}) = 2\underline{a}$